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The Economics of Demographic Uncertainty

Introduction

It is well documented that the future demographic development is uncertain and difficult to predict (see for example Keilman, Cruijsen and Alho 2007, and Alho, Cruijsen and Keilman, 2007). This uncertainty may have important implications both for individual decisions and for economic policy. Several papers in this volume examine how macroeconomic aggregates or public finances are affected by alternative demographic developments under the assumption that these demographic developments are perfectly predicted and anticipated by economic decision makers. The final three papers (Alho and Määttänen, 2007, Jensen and Jørgensen, 2007, and van de Ven and Weale, 2007) differ in that they explicitly model the demographic uncertainty and allow agents to react and take precautionary measures in response to this uncertainty.

A number of important economic topics can only be studied if uncertainty is explicitly modelled. The above mentioned papers take a step in the right direction and contribute by introducing demographic uncertainty to analyze some specific questions. Alho and Määttänen analyze how households respond to the demographic uncertainty in a partial equilibrium setting. Van de Ven and Weale analyze how different generations can share demographic risks on private markets, whereas Jensen and Jørgensen analyze how economic policy may improve risk sharing, and to some extent how economic policy may be affected by demographic shocks. These papers also raise a number of issues that need further study, and they indicate that integrating different elements of their analysis could be important.

Household behaviour and welfare

The first issue I want to address is how demographic uncertainty affects behaviour (as analyzed by Alho and Määttänen) and welfare and the value of insurance (as analyzed by van de Ven and Weale). In particular, it is worth noting that the analysis from settings with income uncertainty is not directly applicable in this setting.[1]

[1] Pratt (1964) analyzes the effects of income uncertainty on welfare, and Kimball (1990) analyzes the effects on behaviour.

Let me use a simplistic analytical framework to illustrate this. Consider an endowment economy where households live for a maximum of two periods. Young households have exogenous income y which can be consumed (c_1) or saved (s). A stochastic fraction ρ of households survive to the second period where they consume (c_2) what was saved in the first period. There is an annuity market so that those who survive share the savings of those who did not survive. Households then maximize expected life-time utility by solving $\max \mathrm{E}\{\mathrm{u}(c_1) + \beta\rho\,\mathrm{u}(c_2)\}$ subject to $c_1+s = y$ and $c_2 = s/\rho$, where β is the discount factor and E is the expectations operator.[2] Note that second-period consumption for the survivors is stochastic if the aggregate survival probability is stochastic.

The solution to the household's problem is characterized by the Euler equation $\mathrm{u}'(c_1) = \beta \mathrm{E}\,\mathrm{u}'(c_2)$. Assume that utility belongs to the standard constant relative risk aversion class of functions, $\mathrm{u}(c) = c^{1-\mu}/(1-\mu)$, and let $\overline{\rho}$ and σ denote the mean and variance of ρ. A second-order expansion of $\mathrm{u}'(c_2)$ around $\overline{\rho}$ implies that the Euler equation can be approximated as

$$\mathrm{u}'(c_1) \approx \beta u'\left(\frac{s}{\overline{\rho}}\right)\left[1+\frac{\sigma^2\mu(\mu-1)}{2\overline{\rho}^2}\right] \tag{1}$$

In the analysis of behaviour under income uncertainty, Kimball (1990) demonstrates that this utility function implies precautionary behaviour for any risk aversion $\mu > 0$, i.e. higher income uncertainty implies higher saving and less consumption in the first period. From (1) we see that higher demographic uncertainty ($\sigma\uparrow$) raises s only if $\mu > 1$. If utility is logarithmic ($\mu = 1$), behaviour does not change in response to changed uncertainty. This result helps explain Alho and Määttänen's finding that households' may not loose much by ignoring uncertainty. The optimal response may actually be to behave as if survival probabilities were non-stochastic.

The analysis of welfare consequences of demographic uncertainty is also different from the standard analysis under income uncertainty. The stochastic variable has a direct impact on utility in the second period, $\beta\rho\mathrm{u}(c_2)$. Note that income uncertainty in the second period only affects c_2. Welfare falls when second-period consumption becomes more volatile if the instantaneous utility function, u, is

[2] For some questions it may be important to guarantee higher longevity raises welfare. Life-time utility can then be specified as $\mathrm{u}(c_1) + \beta\rho\,\mathrm{u}(c_2) + (1-\rho)D$ where D is a sufficiently large negative number.

concave. Mortality risk has somewhat different implications since it affects not only consumption but also the effective discount rate, $\beta\rho$.

Consider for example annuities that reduce the volatility of second-period consumption. The difference between income and mortality risk affects the interpretation of what these annuities do. If the instantaneous utility function is concave in consumption then $\beta\rho\mathrm{u}(c_2)$ is concave in ρ, and households would benefit if the demographic uncertainty could be reduced. The annuities, however, cannot reduce the demographic uncertainty but only its impact on consumption. If annuities succeed in holding c_2 constant, welfare will increase but households will face *more risk* since $\beta\rho\mathrm{u}(c_2)$ will become more volatile.[3] These annuities reduce the convexity but not the volatility of $\beta\rho\mathrm{u}(c_2)$.

This analysis does not only affect the interpretation of what annuities do, but also indicates that the optimal insurance typically does not imply constant consumption. Since the annuities only can affect one of the two channels through which demographic uncertainty affects utility, it is typically not optimal to fully eliminate volatility in that channel. An optimal insurance will instead let the outcome in the channel that can be affected (consumption) counteract the outcome in the other channel, which implies that consumption will vary with the aggregate survival rate.

The scope for insurance

The other issue I want to address is the scope for insurance. In particular I want to focus on two questions. First, what insurance can be provided by private markets and what is the role of the government? Second, how are the possibilities to insure affected by the nature of the stochastic process for the demographic development?

Consider an economy that is populated by overlapping generations of households behaving as in the analytical framework presented above. Since households live for a maximum of two periods, and since all households in a generation face identical risks, there will be no market for insurance. Different generations have lives that overlap a maximum of one period and consequently cannot engage in intertemporal trade. What if we consider a more realistic setting where life spans are longer than two periods? Insurance markets can then arise, but as argued by

[3] To see this, note that fluctuations in ρ have two opposing effects on $\beta\rho\mathrm{u}(c_2)$ in the absence of annuities. A higher ρ raises the number of survivors, a positive welfare effect, but also reduces each survivor’s consumption, a negative welfare effect. By stabilizing consumption, annuities remove the second effect but leave the first effect unmoderated.

Bohn (2005), the scope for private insurance is still limited since much of the uncertainty for a middle-aged generation will be resolved before they can start trading with younger generations. Providing insurance against these long-run risks is therefore best handled by governments that can also engage in trade with generations that have still not been born.

Consider again the analytical framework from above, but assume that the government runs a pay-as-you-go defined benefits pension system. It promises a pension p to all survivors, and it finances this payment by taxing the young. Suppose that aggregate longevity turns out to be unusually high. The government may now reduce the pension p and/or raise tax payments from the young, but it may also let future generations contribute. For example, the government can hold p fixed, raise tax payments of the current young, but also promise higher pensions for the current young so that they are compensated for their tax payments. The government can therefore in principle achieve a better outcome than private markets.

But the scope for government insurance may also be limited. The discussion above implicitly assumed that the demographic shock was temporary, so that longevity increased for today’s old but not (or to a lesser extent) for future generations. If longevity increases for today’s old and all future generations, the strategy described above would require pension payments that increase from generation to generation to compensate for tax payments that also increase, and such strategies will not be feasible in dynamically efficient economies. In general, it will be difficult even for a benevolent government to insure against shocks that affect all present and future generations.

This is how demographic uncertainty often has been modelled in economic papers, including Jensen and Jørgensen (2007) and van de Ven and Weale (2007). More realistically, however, longevity does not fluctuate around a constant mean but increases stochastically over time, as in Alho and Määttänen (2007). Analyzing such non-stationary processes is much more complicated, and new methodological developments will be necessary before such processes can be incorporated in general equilibrium models with private or public insurance.

Concluding remarks

To summarize, I have argued that the impact of demographic uncertainty on welfare and behaviour differs from the impact of income uncertainty, and that there is a need for further theoretical analysis of these issues. It is unlikely that insure against aggregate demographic uncertainty can be provided by the market, but governments can possibly provide such insurance. Furthermore, and maybe more importantly, existing government programs such as pension systems will unavoidably be affected by different demographic outcomes. Future research should explicitly model demographic uncertainty to analyze how public finances are affected when the demographic development is not perfectly anticipated. Examples of other related questions that need further study are: What are the conditions for dynamic efficiency when aggregate demographic developments are uncertain? Can the government (in a dynamically efficient economy) issue debt that with a large probability raises welfare for all future generations? Can the government repay the debt and fulfil pension obligations under all demographic developments?[4]

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[4] In settings with stochastic income or production, Zilcha (1991) and Blanchard and Weil (2001) analyze dynamic efficiency, and Ball, Elmendorf, and Mankiw (1998) examine debt strategies.

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