Appendix to: Why Are Capital Income Taxes So High?

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1 Details of the Optimal Taxation Problem

This appendix provides further details to the Ramsey optimal taxation problem formulated in Section 3 in my paper "Why Are Capital Income Taxes So High?". The structure of this appendix is as follows. First the basic optimization problem from the paper is repeated, but for the special case where optimization is with respect to a single household's welfare. It it is then demonstrated that all households with identical wealth-to-earnings ratios prefer the same policies. Thereafter it is demonstrated that the policy that is the result of optimization with respect to the welfare of a group of households also maximizes the welfare of one particular household. Finally, the optimization problem with the additional constraint that the capital-income tax rate cannot exceed the initial tax rate is presented.

1.1 The Basic Optimization Problem with a Single Optimized Household

Consider the problem described in Section 3 and Appendix A in the paper, but suppose that $I = 1$ and $\omega_1 = 1$ so that a single household's welfare is optimized. That problem then describes the optimal policy Π^* for a household with initial wealth a_0 and productivity z. The equations characterizing the resulting outcome (A.1-A.14 in Appendix A) are repeated here as equations (1) to (14) for convenience.

$$\sum \beta^t \left[u_{Ct} C_t + u_{Ht} H_t\right] = \frac{u_{C0} R_0 A_0}{1+\tau^c} \tag{1}$$

$$\sum \beta^t \left[u_{ct} c_t + u_{ht} h_t\right] = \frac{u_{c0} R_0 a_0}{1+\tau^c} \tag{2}$$

$$C_t + K_{t+1} + G = F\left(K_t, Z H_t\right) + (1-\delta) K_t \tag{3}$$

$$\frac{u_{ct+1}}{u_{ct}} = \frac{u_{Ct+1}}{u_{Ct}} \tag{4}$$

$$\frac{u_{ht}}{z u_{ct}} = \frac{u_{Ht}}{Z u_{Ct}} \tag{5}$$

for $t > 0$

$$W_{ct} + \rho_t u_{cct} u_{Ct+1} - \rho_{t-1} u_{Ct-1} u_{cct}/\beta + \xi_t \left[z u_{cct} u_{Ht} - Z u_{Ct} u_{cht}\right] = 0 \tag{6}$$

$$W_{Ct} - \rho_t u_{CCt} u_{ct+1} + \rho_{t-1} u_{ct-1} u_{CCt}/\beta + \xi_t \left[z u_{ct} u_{CHt} - Z u_{CCt} u_{ht}\right] = \nu_t \tag{7}$$

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$$W_{ht} + \rho_t u_{cht} u_{Ct+1} - \rho_{t-1} u_{Ct-1} u_{cht}/\beta + \xi_t \left[z u_{cht} u_{Ht} - Z u_{Ct} u_{hht} \right] = 0 \tag{8}$$

$$W_{Ht} - \rho_t u_{CHt} u_{ct+1} + \rho_{t-1} u_{ct-1} u_{CHt}/\beta + \xi_t \left[z u_{ct} u_{HHt} - Z u_{CHt} u_{ht} \right] = -\nu_t Z F_{Lt} \tag{9}$$

for $t = 0$

$$W_{c0} + \rho_0 u_{cc0} u_{C1} + \xi_0 \left[z u_{cc0} u_{H0} - Z u_{C0} u_{ch0} \right] = \frac{\lambda u_{cc0} R_0 a_0}{1 + \tau^c} \tag{10}$$

$$W_{C0} - \rho_0 u_{CC0} u_{c1} + \xi_0 \left[z u_{c0} u_{CH0} - Z u_{CC0} u_{h0} \right] = \nu_0 + \frac{\Lambda u_{CC0} R_0 A_0}{1 + \tau^c} \tag{11}$$

$$W_{h0} + \rho_0 u_{ch0} u_{C1} + \xi_0 \left[z u_{ch0} u_{H0} - Z u_{C0} u_{hh0} \right] = \frac{\lambda u_{ch0} R_0 a_0}{1 + \tau^c} \tag{12}$$

$$\begin{aligned} & W_{H0} - \rho_0 u_{CH0} u_{c1} + \xi_0 \left[z u_{c0} u_{HH0} - Z u_{CH0} u_{h0} \right] \\ & = -\nu_0 Z F_{L0} + \frac{\lambda u_{c0} R_{H0} a_0 + \Lambda \left(u_{CH0} R_0 + u_{C0} R_{H0} \right) A_0}{1 + \tau^c} \end{aligned} \tag{13}$$

and finally

$$\beta \nu_{t+1} \left(F_{Kt+1} + 1 - \delta \right) = \nu_t \tag{14}$$

1.2 Optimal Policy is Determined by Wealth-to-Earnings Ratio

Proposition 4 states that if the policy Π^* solves the optimization problem when the welfare of a single household ($I = 1$) with initial state $\bar{s} = (\bar{z}, \bar{a}_0)$ is considered, then this policy is also optimal for all households with the same ratio $\bar{a}_0/\bar{z}^{1+\gamma}$.

Suppose that $\left(\bar{\Lambda}, \bar{\lambda}, \left\{C_t, H_t, K_{t+1}, \bar{c}_t, \bar{h}_t, \bar{\nu}_t, \bar{\rho}_t, \bar{\xi}_t\right\}_t\right)$ solves (1) to (14) when the initial state is $\bar{s}$. To demonstrate that Lemma 4 holds, I will construct another set of variables, $\left(\hat{\Lambda}, \hat{\lambda}, \left\{C_t, H_t, K_{t+1}, \hat{c}_t, \hat{h}_t, \hat{\nu}_t, \hat{\rho}_t, \hat{\xi}_t\right\}_t\right)$ that solves (1) to (14 when the initial state is $\hat{s} = \left(\hat{z} = \alpha \bar{z}, \hat{a}_0 = \alpha^{1+\gamma} \bar{a}_0\right)$. Since this solution has the same aggregate outcome (i.e. the same C, H, and K) it is implemented by the same policy Π^*.

Recall that the first-order condition for labor supply is

$$h_t = \left[\frac{\left(1 - \tau_t^h\right) w_t z}{\zeta \left(1 + \tau^c\right)} \right]^\gamma .$$

This directly implies that $\hat{h}_t = \alpha^\gamma \bar{h}_t$ under policy Π^*.

Note first that equations (1) and (3) are unaffected when we solve for household $\hat{s}$ rather than household $\bar{s}$. Second, I claim that $\hat{c}_t = \alpha^{1+\gamma} \bar{c}_t$. Note that $\hat{c}_t = \alpha^{1+\gamma} \bar{c}_t$ and $\hat{h}_t = \alpha^\gamma \bar{h}_t$ imply that

$$\hat{u}_{ct} = \alpha^{-\mu(1+\gamma)} \bar{u}_{ct},$$

$$\hat{u}_{ht} = \alpha^{1-\mu(1+\gamma)} \bar{u}_{ht},$$

$$\hat{u}_{cct} = \alpha^{-(\mu+1)(1+\gamma)} \bar{u}_{cct},$$

$$\hat{u}_{cht} = \alpha^{-\mu(1+\gamma)-\gamma} \bar{u}_{cht},$$

and

$$\hat{u}_{hht} = \alpha^{-\mu(1+\gamma)+1-\gamma} \bar{u}_{hht}.$$

Using also $\hat{a}_0 = \alpha^{1+\gamma}\bar{a}_0$, equation (2) for household $\hat{s}$ can then be written as

$$\begin{aligned} \sum \beta^t \left[\alpha^{-\mu(1+\gamma)}\bar{u}_{ct}\alpha^{1+\gamma}\bar{c}_t + \alpha^{1-\mu(1+\gamma)}\bar{u}_{ht}\alpha^{\gamma}\bar{h}_t\right] &= \frac{\alpha^{-\mu(1+\gamma)}\bar{u}_{c0}R_0\alpha^{1+\gamma}\bar{a}_0}{1+\tau^c} \\ &\Downarrow \\ \sum \beta^t \left[\bar{u}_{ct}\bar{c}_t + \bar{u}_{ht}\bar{h}_t\right] &= \frac{\bar{u}_{c0}R_0\bar{a}_0}{1+\tau^c} \end{aligned}$$

which is equation (2) for the $\bar{s}$ household. That is, if $\left\{\bar{c}_t, \bar{h}_t\right\}$ fulfils the implementability constraint for household $\bar{s}$, then $\left\{\hat{c}_t = \alpha^{1+\gamma}\bar{c}_t, \hat{h}_t = \alpha^{\gamma}\bar{h}_t\right\}$ fulfils the implementability constraint for household $\hat{s}$. Similar observations are immediate for equations (4) and (5).

We next see that equation (6) holds for $\left\{\hat{c}_t, \hat{h}_t\right\}$ if it holds for $\left\{\bar{c}_t, \bar{h}_t\right\}$ and if $\hat{\lambda} = \bar{\lambda}$, $\hat{\rho}_t = \alpha^{1+\gamma}\bar{\rho}_t$ and $\hat{\xi}_t = \alpha^{\gamma}\bar{\xi}_t$. This in turn implies that (7) holds if $\hat{\Lambda} = \alpha^{(1-\mu)(1+\gamma)}\bar{\Lambda}$ and $\hat{\nu}_t = \alpha^{(1-\mu)(1+\gamma)}\bar{\nu}_t$. It is then straightforward to verify that also (8) to (14) are satisfied, which confirms that Π^* is also the optimal policy for household $\hat{s}$.

1.3 Many Optimized Households and a Stand-In Household

Consider a policy Π^* that maximizes the social welfare function

$$W(\Pi^*) = \sum_i \omega_i \sum_{t=0}^{\infty} \beta^t u\left(c_t(s_i), h_t(s_i)\right)$$

where the first sum is over households in the economy, and ω_i is the weight put on household i's welfare.

Proposition 5 states that the policy Π^* is also optimal for a stand-in household with initial allocations $(\bar{a}_0, \bar{z} = 1)$.

Suppose that $\left(\Lambda, \{C_t, H_t, K_{t+1}, \nu_t\}_t, \{\lambda_i, \{c_{it}, h_{it}, \rho_{it}, \xi_{it}\}_t\}_i\right)$ solves the system of first-order equations (A.1)-(A.14) in Appendix A in the paper. To demonstrate that Proposition 5 holds, I will construct another set of variables, $\left(\bar{\Lambda}, \bar{\lambda}, \left\{C_t, H_t, K_{t+1}, \bar{c}_t, \bar{h}_t, \bar{\nu}_t, \bar{\rho}_t, \xi_t\right\}_t\right)$ that solves (1) to (14) when the optimization is with respect to the welfare of a single stand-in household with initial state $\bar{s} = (\bar{z} = 1, \bar{a}_0)$ for some $\bar{a}_0$. Since this solution has the same aggregate outcome as the solution to the many-households problem (i.e. the same C, H, and K) it is implemented by the same policy Π^*.

From the first-order condition for optimal labor choices we see that $\bar{h}_t = z_i^{-\gamma}h_{it}$ under policy Π^*. Define $x_i = \left(u_{ci0}/\bar{u}_{c0}\right)^{1/\mu}$. It then follows that

$$\bar{c}_t - \zeta\frac{\bar{h}_t^{1+1/\gamma}}{1+1/\gamma} = x_i\left(c_{it} - \zeta\frac{h_{it}^{1+1/\gamma}}{1+1/\gamma}\right)$$

and

$$\bar{c}_t = x_i c_{it} + \left(z_i^{-1-\gamma} - x_i\right)\zeta\left(h_{it}\right)^{1+1/\gamma}/\left(1+1/\gamma\right) \tag{15}$$

Note now that

$$\bar{u}_{ct} = x_i^{-\mu}u_{cit}, \tag{16}$$

$$\bar{u}_{ht} = z_i^{-1}x_i^{-\mu}u_{hit}, \tag{17}$$

$$u_{cct} = x_i^{-\mu-1}u_{ccit} \tag{18}$$

$$\bar{u}_{cht} = z_i^{-1}x_i^{-\mu-1}u_{chit}, \tag{19}$$

and

$$\bar{u}_{hht} = z_i^{-2} x_i^{-\mu-1} \left[u_{hht}^A + \left(z^{1+\gamma} x_i - 1 \right) \frac{u_{hit}}{\gamma h_{it}} \right]. \tag{20}$$

The solution to the many-households problem satisfies (A.11) while the solution to the one-household problem must satisfy (11), i.e.:

$$\bar{\Lambda} \left(u_{C0} + u_{CC0} C_0 + u_{CH} H_0 \right) - \bar{\rho}_0 u_{CC0} \bar{u}_{c1} + \bar{\xi}_0 \left(\bar{z} \bar{u}_{c0} u_{CH0} - Z u_{CC0} \bar{u}_{h0} \right) = \bar{\nu}_0 + \frac{\Lambda u_{CC0} R_0 A_0}{1+\tau^c}. \tag{21}$$

Using (16) and (17), (A.11) can be rewritten as

$$\Lambda \left(u_{C0} + u_{CC0} C_0 + u_{CH} H_0 \right) - u_{CC0} \bar{u}_{c1} \sum_i x_i^\mu \rho_{i0} +$$
$$\left(\bar{u}_{c0} u_{CH0} - Z u_{CC0} \bar{u}_{h0} \right) \sum_i z_i x_i^\mu \xi_{i0} = \nu_0 + \frac{\Lambda u_{CC0} R_0 A_0}{1+\tau^c}.$$

Thus, if p is some constant and we let $\bar{\Lambda} = p^{-1}\Lambda$, $\bar{v}_t = p^{-1} v_t$, $\bar{\rho}_t = p^{-1} \sum_i x_i^\mu \rho_{it}$, and $\bar{\xi}_t = p^{-1} \sum_i z_i x_i^\mu \xi_{it}$ we see that (21) holds. We also see that these multipliers together with (A.7), (A.9), and (A.13) imply that the corresponding one-household equations (7), (9), and (13) hold. Note that x_i is unknown until we have determined $\bar{a}_0$ and identified the stand-in household. Define therefore $y_i = \left(u_{ci0}/u_{c10} \right)^{1/\mu}$ and note that all y_i are known when we have solved the many-household problem. Note also that $x_i = y_i x_1$ so that the multipliers $\bar{\rho}$ and $\bar{\xi}$ can be rewritten as $\bar{\rho}_t = p^{-1} x_1^\mu \sum_i y_i^\mu \rho_{it}$, and $\bar{\xi}_t = p^{-1} x_1^\mu \sum_i z_i y_i^\mu \xi_{it}$, where x_1 is still unknown.

The solution to the many-households problem also satisfies (A.10) while the solution to the one-household problem must satisfy (10):

$$\bar{u}_{c0} + \bar{\lambda} \left(\bar{u}_{c0} + \bar{u}_{cc0} \bar{c}_0 + \bar{u}_{ch0} \bar{h}_0 \right) + \bar{\rho}_0 \bar{u}_{cc0} u_{C1} + \bar{\xi}_0 \left(\bar{u}_{cc0} u_{H0} - Z u_{C0} \bar{u}_{ch0} \right) = \frac{\bar{\lambda} \bar{u}_{cc0} R_0 \bar{a}_0}{1+\tau^c}. \tag{22}$$

Now multiply each (A.10) by x_i^{-1} and then sum over all households to get

$$\sum_i x_i^{-1} \omega_i u_{ci0} + \sum_i x_i^{-1} \lambda_i \left(u_{ci0} + u_{cci0} c_{i0} + u_{chi0} h_{i0} \right) + \sum_i x_i^{-1} \rho_{i0} u_{cci0} u_{C1} +$$
$$\sum_i x_i^{-1} \xi_{i0} \left(z_i u_{cci0} u_{H0} - Z u_{C0} u_{chi0} \right) = \frac{\sum_i x_i^{-1} \lambda_i u_{cci0} R_0 a_{i0}}{1+\tau^c}.$$

Using (15) to (19) this can be rewritten as

$$\bar{u}_{c0} \sum_i x_i^{\mu-1} \omega_i + \left(\bar{u}_{c0} + \bar{u}_{cct} \bar{c}_0 + \bar{u}_{ch0} \bar{h}_0 \right) \sum_i x_i^{\mu-1} \lambda_i + \bar{u}_{cc0} u_{C1} \sum_i x_i^\mu \rho_{i0} +$$
$$\left(\bar{u}_{cc0} u_{H0} - Z u_{C0} \bar{u}_{ch0} \right) \sum_i x_i^\mu z_i \xi_{i0} = \frac{\bar{u}_{cc0} R_0 \sum_i x_i^\mu \lambda_i a_{i0}}{1+\tau^c}. \tag{23}$$

Using our previous conclusions about the multipliers we can write (23) as

$$\bar{u}_{c0} x_1^{\mu-1} \sum_i y_i^{\mu-1} \omega_i + \left(\bar{u}_{c0} + \bar{u}_{cct} \bar{c}_0 + \bar{u}_{ch0} \bar{h}_0 \right) \sum_i x_i^{\mu-1} \lambda_i + \bar{u}_{cc0} u_{C1} p \bar{\rho}_t +$$
$$\left(\bar{u}_{cc0} u_{H0} - Z u_{C0} \bar{u}_{ch0} \right) p \bar{\xi}_t = \frac{\bar{u}_{cc0} R_0 \sum_i x_i^\mu \lambda_i a_{i0}}{1+\tau^c}. \tag{24}$$

Now let $p = x_1^{\mu-1} \sum_i y_i^{\mu-1} \omega_i$ and divide by p on both sides in (24) to get

$$\bar{u}_{c0} + \left(\bar{u}_{c0} + \bar{u}_{cct} \bar{c}_0 + \bar{u}_{ch0} \bar{h}_0 \right) p^{-1} x_1^{\mu-1} \sum_i y_i^{\mu-1} \lambda_i + \bar{u}_{cc0} u_{C1} \bar{\rho}_t +$$
$$\left(\bar{u}_{cc0} u_{H0} - Z u_{C0} \bar{u}_{ch0} \right) \bar{\xi}_t = \frac{\bar{u}_{cc0} R_0 x_1^\mu \sum_i y_i^\mu \lambda_i a_{i0}}{1+\tau^c}.$$

Let now

$$
\begin{aligned}
\bar{\lambda} &= p^{-1} x_1^{\mu-1} \sum_i y_i^{\mu-1} \lambda_i \\
&= \left(\sum_i y_i^{\mu-1} \omega_i \right)^{-1} \left(\sum_i y_i^{\mu-1} \lambda_i \right). \qquad (25)
\end{aligned}
$$

Note that all variables on the right hand side in (25) are known when the many-households problem has been solved. As a result $\bar{\lambda}$ can be calculated and it will be possible to identify the stand-in household as described below.

We now see that (10) holds if

$$\bar{\lambda} \bar{a}_0 = x_1^{\mu} \sum_i y_i^{\mu} \lambda_i a_{i0}$$

which is the case if we let

$$
\begin{aligned}
\bar{a}_0 &= \bar{\lambda}^{-1} x_1^{\mu} \sum_i y_i^{\mu} \lambda_i a_{i0} \\
&= x_1^{\mu} \left(\sum_i y_i^{\mu-1} \omega_i \right) \left(\sum_i y_i^{\mu-1} \lambda_i \right)^{-1} \left(\sum_i y_i^{\mu} \lambda_i a_{i0} \right).
\end{aligned}
$$

We can also verify that this multiplier $\bar{\lambda}$ and initial wealth position $\bar{a}_0$ together with (A.6), (A.8), and (A.12) imply that (6), (8), and (12) hold.

Identifying the stand-in household: The expression for $\bar{\lambda}$ in (25) demonstrates that this multiplier on the stand-in household's implementability constraint is a weighted average of the multipliers for the optimized households in the many-households problem. If we consider solutions to the one-household problem for different initial wealth positions $\hat{a}_0$ (but with fixed productivity $\bar{z}$), the resulting multiplier $\hat{\lambda}$ is continuous in $\hat{a}_0$ and it will be possible to find the unique $\hat{a}_0$ such that $\hat{\lambda} = \bar{\lambda}$ as specified in (25). After finding this $\hat{a}_0$ that implies $\bar{\lambda}$ we know $\bar{a}_0 = \hat{a}_0$ and can therefore calculate x_1. This in turn implies that we can calculate all variables in $\left(\bar{\Lambda}, \bar{\lambda}, \left\{C_t, H_t, K_{t+1}, \bar{c}_t, \bar{h}_t, \bar{\nu}_t, \bar{\rho}_t, \xi_t\right\}_t\right)$ as specified above. The calculations above demonstrate that these variables fulfil the first-order conditions for the stand-in household, and thus that optimization with respect to this household will result in the same optimal policy as optimization with respect to the welfare of the group of households initially considered.

Practical implications: In principle we can thus find $\bar{\lambda}$ by solving the many-households problem and then constructing the stand-in household as demonstrated above. The problem motivating the construction of a stand-in household is however that it is numerically difficult to solve the many-household problem. Proposition 5 instead demonstrates that a stand-in household exists. Knowing this, and knowing that only the wealth-to-earnings ration $a_0/z^{1+\gamma}$ affects the optimal policy, we can conclude that the solution to the many-households optimization problem can be found numerically by searching for the initial wealth position $\hat{a}_0$ that that maximizes the welfare of these households when the policy is optimized for a stand-in household with initial state $(\bar{z} = 1, \hat{a}_0)$.

1.4 Constraints on the Capital-Income Tax

We first consider the case when the tax rate is restricted not to exceed the initial value, i.e. $\tau_t^k \leq \tau_{ss}^k$.[1] The solution will then have the property that $\tau_t^k = \tau_{ss}^k$ for $t \leq t^*$, and $\tau_t^k < \tau_{ss}^k$ for $t > t^*$. I therefore guess that $t^* = \hat{t}$, solve the problem with the constraint $\tau_t^k = \tau_{ss}^k$ for $t \leq \hat{t}$ imposed, and then check if $\tau_{\hat{t}+1}^k < \tau_{ss}^k$. If so, and if this is the smallest $\hat{t}$ where this is the case, then I conclude that $t^* = \hat{t}$.

[1] Note that the specification in the paper already contains the restriction that the captil tax rate in the first period is equal to the initial tax rate, $\tau_0^k = \tau_{ss}^k$.

After guessing $\hat{t}$, the problem is to solve the same problem as described in Section 3 in the paper but with the additional restriction on capital taxes,

$$\tau_t^k = \tau_{ss}^k \text{ for } t \leq \hat{t}.$$

Using the equilibrium condition $F_{Kt} = r_t + \delta$ and the definition of R_t, note that the tax rate can be written as

$$\tau_t^k = 1 + \frac{1 - R_t}{F_{Kt} - \delta}.$$

The Euler equation further implies that $R_{t+1} = u_c(C_t, H_t) / [\beta u_c(C_{t+1}, H_{t+1})]$ so that the constraint on capital taxes can be written as

$$1 + \frac{1 - u_c(C_t, H_t) / [\beta u_c(C_{t+1}, H_{t+1})]}{F_{Kt+1} - \delta} = \tau_{ss}^k \text{ for } t \leq \hat{t}.$$

Let then[2]

$$\Psi(C_{t-1}, H_{t-1}, C_t, H_t, K_t) = 1 - \frac{u_c(C_{t-1}, H_{t-1})}{\beta u_c(C_t, H_t)} + \left(1 - \tau_{ss}^k\right)(F_{Kt} - \delta).$$

so that the constraint on capital taxes can be written as

$$\Psi(C_{t-1}, H_{t-1}, C_t, H_t, K_t) = 0 \text{ for } 0 < t \leq \hat{t}. \tag{26}$$

The Lagrangian to this problem is then

$$\begin{aligned}
\pounds \quad = \quad & \sum_{t=0}^{\infty} \beta^t W(c_t, C_t, h_t, H_t, \lambda, \Gamma) - (\lambda u_{c0} a_0 + \Lambda u_{C0} A_0) \frac{R_0}{1 + \tau^c} + \\
& \sum_{t=0}^{\infty} \beta^t \nu_t [F(K_t, ZH_t) + (1 - \delta) K_t - C_t - K_{t+1} - G] + \\
& \sum_{t=0}^{\infty} \beta^t \rho_t [u_{ct} u_{Ct+1} - u_{Ct} u_{ct+1}] + \\
& \sum_{t=0}^{\infty} \beta^t \xi_t [z u_{ct} u_{Ht} - Z u_{Ct} u_{ht}] + \\
& \sum_{t=1}^{\hat{t}} \beta^t \psi_t \Psi(C_{t-1}, H_{t-1}, C_t, H_t, K_t)
\end{aligned}$$

where ν, ρ, ξ, and ψ are Lagrange multipliers. The first order conditions for c_t, h_t, Λ, λ, ν_t, ρ_t, ξ_t are unaffected by the restriction on the capital-income taxes but the first order conditions for C_t, and H_t become (for $t > 0$)

$$W_{Ct} - \rho_t u_{CCt} u_{ct+1} + \rho_{t-1} u_{ct-1} u_{CCt} / \beta + \xi_t [z u_{ct} u_{CHt} - Z u_{CCt} u_{ht}] + \beta \psi_{t+1} \Psi_{C-t+1} + \psi_t \Psi_{Ct} = \nu_t$$

$$\begin{gathered}
W_{Ht} - \rho_t u_{CHt} u_{ct+1} + \rho_{t-1} u_{ct-1} u_{CHt} / \beta + \xi_t [z u_{ct} u_{HHt} - Z u_{CHt} u_{ht}] + \\
\beta \psi_{t+1} \Psi_{H-t+1} + \psi_t \Psi_{Ht} = -\nu_t Z F_{Lt}
\end{gathered}$$

if we let $\psi_t = 0$ for $t > \hat{t}$, and (for $t = 0$)

$$W_{C0} - \rho_0 u_{CC0} u_{c1} + \xi_0 [z u_{c0} u_{CH0} - Z u_{CC0} u_{h0}] + \beta \psi_1 \Psi_{C-1} = \nu_0 + \frac{\lambda u_{CC0} R_0 A_0}{1 + \tau^c}$$

[2] I will let Ψ_{C-} and Ψ_{H-} denote the derivative of Ψ with respect to its first two arguments.

$$W_{H0} - \rho_0 u_{CH0} u_{c1} + \xi_0 \left[z u_{c0} u_{HH0} - Z u_{CH0} u_{h0} \right] + \beta \psi_1 \Psi_{H-1} + \psi_t \Psi_{Ht}$$
$$= -\nu_0 Z F_{L0} + \frac{\lambda u_{c0} R_{H0} a_0 + \Lambda \left(u_{CH0} R_0 + u_{C0} R_{H0} \right) A_0}{1 + \tau^c}$$

and the first order conditions for K_{t+1} becomes

$$\beta \left[\nu_{t+1} \left(F_{Kt+1} + 1 - \delta \right) + \psi_{t+1} \Psi_{Kt+1} \right] = \nu_t,$$

while (26) is the first order condition for the multiplier ψ_t.